On the Dynamic Detection of Interchangeability in Finite Constraint Satisfaction Problems

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We investigate techniques that detect, dynamically during search, *undeclared* symmetries in the form of interchangeability (Freuder'91) in Constraint Satisfaction Problems, with the long-term goal of drawing a compact landscape of the solution space of a given CSP instance. As a first step towards our goal, we propose a new algorithm for dynamically computing interchangeability during backtrack search and demonstrate how it enhances the performance of search.

A technique for exploiting interchangeability during search, which we call FC-NI_C (i.e., forward checking with neighborhood interchangeability according to one constraint), is described by Haselböck (1993). NI_C sets are computed in a pre-processing step prior to search and yield a *static* bundling of the solution space. Since Freuder noted that the *dynamic* computation of interchangeability sets during search yields more opportunities for interchangeability, it is natural to suspect that it also yields better bundling. We design, implement and test a new *dynamic* bundling strategy that we call FC-DNPI. While it was so far believed that the dynamic computation of interchangeability would be too costly to implement during search, we prove that FC-DNPI (i.e., forward checking with dynamic neighborhood partial interchangeability) requires *no more constraint checks than those necessary for performing forward checking.* This strategy computes the DNPI sets using the Joint Discrimination Tree (JDT) of Choueiry and Noubir (1998), which is a generalization of the discrimination tree (DT) of Freuder (1991). Further, we uncover the never-stated relation between interchangeability and the Cross Product Representation (CPR) of Hubbe and Freuder (1992) as being almost equivalent: Search with CPR (FC-CPR) and FC-DNPI yield exactly the same bundles and constraint checks while visiting more nodes in the tree.

We provide theoretical guarantees for the performance of FC-DNPI by comparing it with FC, FC-NI_C, and FC-CPR in terms of the numbers of nodes visited, constraint checks, and generated bundles. Then we provide empirical evidence of the advantages of FC-DNPI in terms of these criteria and also CPU time in the following settings: finding all solutions or the first solution (bundle) to the CSP under static, dynamic variable, and dynamic variable-value orderings. Our tests are performed in the most adversary conditions, i.e. randomly generated problems whose degree of embedded interchangeability is finely controlled over a complete range. We establish that dynamic bundling is orthogonal to, and can only benefit from, traditional enhancements to backtrack search such as strategies for dynamic ordering and for maintaining-arc consistency.