Evaluating Consistency Algorithms for Temporal Metric Constraints

Yang Shi, Anagh Lal, and Berthe Y. Choueiry
Constraint Systems Laboratory • Computer Science & Engineering • University of Nebraska-Lincoln • yshi, alal, choueiry@cse.unl.edu

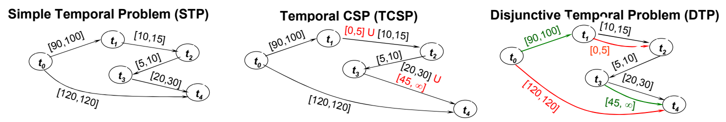
Summary

Focus: Networks of temporal metric constraints
Task: Evaluating the performance of algorithms for
- Determining the consistency of the Simple Temporal Problem (STP)
- Finding the minimal network of the Temporal Constraint Satisfaction Problem (TCSP)

Future: Enhance triangulation-based algorithms with incrementality

Networks of Temporal Metric Constraints

Temporal constraint network: a graph G=(V, E, I) where
- V: set of vertices representing time points t_i
- E: set of directed edges representing constraints between two time points t_i & t_j
- I: set of constraint labels for the edges. A label is a set of intervals and an interval [*a*, *b*] denotes a constraint of bounded differences ($a \le t_j - t_i \le b$)



Constraint: *One interval per edge* — *A disjunction of intervals per edge* — *A disjunction of intervals from edges*

Minimal network: Make labels of binary constraints as tight as possible
Solution: Find a value for each variable satisfying all temporal constraints
Consistency: Determine whether a solution exists

	STP	TCSP	DTP
Minimal network	P	NP-hard	NP-hard
Consistency	P	NP-complete	NP-complete

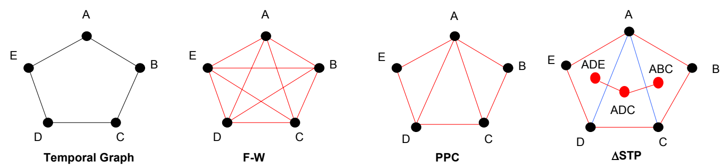
Problem complexity

Algorithms for the STP

Determining consistency
- Directional Path Consistency (DPC)
- Bellman-Ford (BF), single-source shortest paths
- Incremental version of Bellman-Ford (incBF) [Cesta & Oddi, TIME 96]

Determining consistency & finding minimal network
- Floyd-Warshall (F-W), all-pairs shortest paths
- Partial Path Consistency (PPC) [Bliek & Haroud, IJCAI 99]
- ΔSTP: an improvement of PPC [Xu & Choueiry, TIME 03]

Properties & advantages of ΔSTP
ΔSTP considers the temporal graph as composed of triangles instead of edges

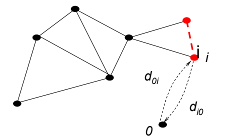


Constraint graphs of different solving strategies

1. A finer version of PPC.
2. Cheaper than PPC and F-W.
3. Guarantees the minimal network.
4. Automatically decomposes the graph into its bi-connected components:
   - binds effort in size of largest component.
   - allows parallellization.
5. Best known algorithm for computing the minimal network of an STP

An incremental version of BF (incBF):
When adding a constraint, incBF visits only nodes whose distance to origin is modified:

1. Allows dynamic updates for both constraint posting & retraction.
2. Localizes effects of change.
3. Determines consistency of STP *by does not yield the minimal network.*
4. Can detect inconsistency much earlier than BF by detecting negative cycles ($d_{0i} + d_{i0} < 0$).
5. Is useful for TCSP: incrementality is useful for checking the consistency of STPs in the search tree of the meta-CSP.



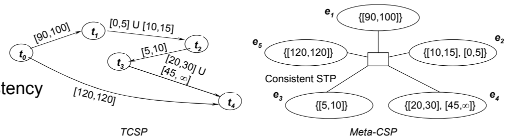
Point 0 is the source added to the constraint graph. incBF updates only affected distances, and detects inconsistency when re-visiting a node.

Comparing the above strategies:

	Graph	Complexity	Consistency	Minimality
F-W	Complete	$\Theta(n^3)$	Yes	Yes
DPC	Triangulated	$O(nW(d)^2)$ very cheap	Yes	No
PPC	Triangulated	$O(n^3)$ Usually cheaper than F-W/PPC	Yes	Yes
ΔSTP	Triangulated	Always cheaper than PPC	Yes	Yes
BF/incBF	Source point is added	$O(en)$	Yes	No

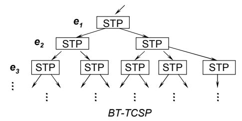
Solving the TCSP [Dechter et al. AIJ 91]

TCSP is formulated as a meta-CSP
- Variables: edges of the constraint network
- Domains of variables: edge labels in the constraint network
- A unique global constraint: checking consistency of an STP



The minimal network of the TCSP can be found by computing *all the solutions* to the meta-CSP

When using backtrack search for finding all the solutions to the meta-CSP (BT-TCSP), every node in the search tree is an STP to be checked for consistency
→ An exponential number of STPs to be considered!



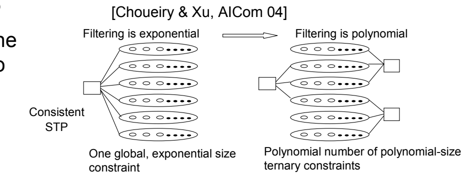
Improving Search for the TCSP [Xu & Choueiry CP 03]

Improve the performance of BT-TCSP:
- **ΔAC:** a consistency filtering algorithm for reducing the size of TCSP.
- Exploit the topology of the constraint graph:
  - **AP**: using articulation points
  - **NewCyc**: a heuristic for avoiding unnecessary checking of STPs at every node.
  - **EdgeOrd**: a variable ordering heuristic.

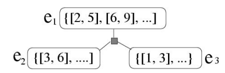
ΔAC: A new algorithm for filtering TCSP

ΔAC removes inconsistent intervals from the domain of the variables of the meta-CSP to reduce the size of meta-CSP:
- In a pre-processing step (implemented)
- In a look-ahead strategy (to be tested)

[Choueiry & Xu, AICom 04]



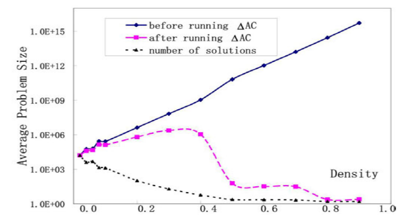
ΔAC checks combinations of 3 intervals:



✓ [2,5] composed with [1, 3] intersects with [3, 6]
✓ [1,3] composed with [3, 6] intersects with [2, 5]
★ [3,6] composed with [2, 5] does not intersect with [1, 3]

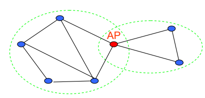
ΔAC removes [1, 3] from domain of e_3.

Advantages of **ΔAC:**
- It is effective, especially under high density.
- It is sound, cheap $O(n\,|E|\,k3)$, may be optimal.
- It uncovers a phase transition in TCSP.



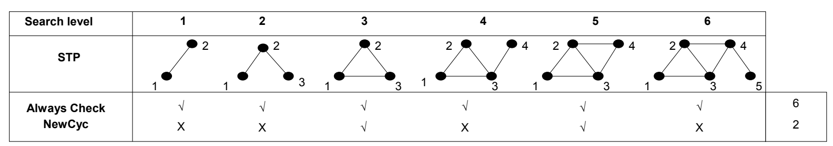
Reduction of problem size of the TCSP

Articulation Points (AP) exploits the topology of the graph
- Decomposes the graph into bi-connected components.
- Solves each of them independently.
- Binds the total cost by the size of largest component.



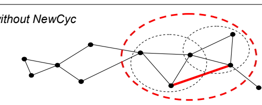
Decomposition Using Articulation Point

New cycle check (NewCyc) eliminates unnecessary STP-consistency checks
- Checks presence of new cycles $O(|E|)$.
- Checks consistency only when a new cycle is added.
- Does not affect number of nodes visited in BT-TCSP.



Number of STP checks with and without NewCyc

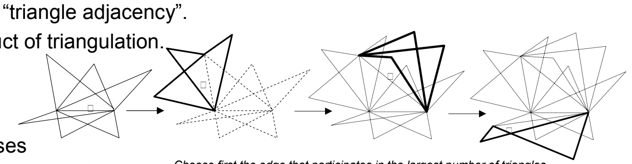
Advantages of **NewCyc**:
- Reduces effort of consistency checking.
- Restricts effort to new bi-connected component.



Checks the consistency of only the newly formed biconnected component

Edge Ordering (EdgeOrd): a variable ordering heuristic in BT-TCSP
- Orders the edges using "triangle adjacency".
- Priority list is a by-product of triangulation.

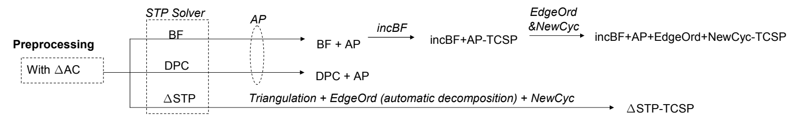
Advantages of **EdgeOrd**
- Localizes backtracking.
- Automatically decomposes the constraint graph ⇒ no need for AP.



Choose first the edge that participates in the largest number of triangles, then consider in priority the edges of the triangles where it appears

Experiments

We tested the following combinations:

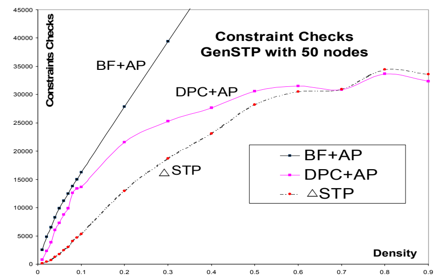


Random generators of STP & TCSP:
- Generators take as input:
  1. Number of time points of the TCSP
  2. Constraint density
  3. (Number of intervals per edge)
  4. Percentage of problems guaranteed consistent
- Note that size of meta-CSP is exponential in the number of time points

Measured:
CPU time, NV number of nodes visited (for TCSP), & CC number of constraint checks

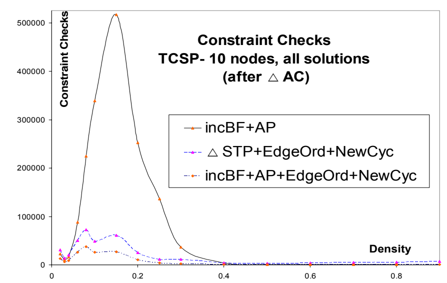
Results of Empirical Evaluations

Experiments on the STP:
- 50-node STP, density in [2%, 90%], 100 samples per point
- ΔSTP results in the minimal network & dominates all others
- Cost of BF increases linearly with density (bounded by *O(en)*, where *n* and *e* are respectively the number of nodes and the number of edges in the graph).



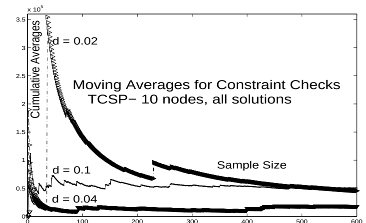
Constraint checks for selected STP solvers

Experiments on TCSP (all solutions):
- 10-node TCSP, density in [2%, 90%], 600 samples per point
- Search enhanced with ΔAC, AP, NewCyc, EdgeOrd



Constraint checks for selected TCSP solvers (finding minimal network of the TCSP)

For small density values (<0.1), values of results were instable. We increased number of samples up to 600 samples per point:



For small values of d, the average of CC, average of CC is not stable when the sample size is less than 400.

d	Δ STP CCx10³	incBF CCx10³	Gain CCx10³ LL	Average	UL
0.02	45.61	14.77	5.39	30.84	56.29
0.04	17.51	7.56	5.06	9.95	14.84
0.06	51.66	24.30	3.45	27.35	51.24
0.08	83.38	50.74	4.86	32.63	60.41
0.10	50.31	26.24	20.29	24.07	27.84
0.15	75.92	37.61	20.52	38.30	56.08
0.20	28.09	12.03	10.74	16.06	21.38

Average CC gain of the best strategy and its lower limit (LL) and upper limit (UL) with 95% confidence.

Conclusions

For STP: ΔSTP outperforms all others
For TCSP:
- incBF outperforms ΔSTP
- EdgeOrd & NewCyc always beneficial

Future: exploit incrementality

Algorithm	Performance Ranking STP	TCSP
FW + AP	worse	worse
DPC + AP	better	OK
BF + AP	OK	-
Δ STP	best	-
incBF + AP	good	good
Δ STP + EdgeOrd + NewCyc	-	better
incBF + AP + EdgeOrd + NewCyc	-	best

References

Cesta & Oddi. Gaining Efficiency and Flexibility in the Simple Temporal Problem. TIME 96
Choueiry & Xu. An Efficient Consistency Algorithm for the Temporal Constraint Satisfaction Problem. AICom 2004.
Dechter, Meiri, & Pearl. Temporal Constraint Networks. AIJ 91.
Stergiou & Koubarakis. Backtracking Algorithms for Disjunctions of Temporal Constraints. AIJ 2000.
Xu & Choueiry. A New Efficient Algorithm for Solving the Simple Temporal Problem. TIME 2003.
Xu & Choueiry. Improving Backtrack Search for Solving the TCSP. CP 2003.

Support: Layman award, NASA-Nebraska grant, NSF CAREER Award #0133568

University of Nebraska Lincoln