Introduction of the Minfill Heuristic

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1 Related Work

The Karypis Lab has developed a large library\(^1\) of graph partitioning algorithm (MeETIS, ParMETIS, hMETIS). This library is particularly well suited for large graphs. It is commonly used by the UAI community for tree decomposition, and should be checked before doing any implementation work.

[1] presents a linear time algorithm for minimal elimination ordering approximation in planar graphs.

2 An \( O(n^4) \) Implementation for Minfill

Below, we describe an implementation of the well-known minfill algorithm, which is described in [2]. In our implementation, we store in \( fcount[x] \) the number of fill edges that need to be added to the graph when the vertex \( x \) is removed.

\(^1\)http://glaros.dtc.umn.edu/gkhome/views/metis
Algorithm 1: MinFill(G)

Input: A graph $G = (V, E)$, where $|V| = n$.
Output: Perfect elimination order $\sigma$

1. FillCount(G)
for $i = 1$ to $n$ do

2. $v \leftarrow$ the vertex in $G$ with the smallest value of $fcount$
3. $\sigma[i] \leftarrow v$
4. AddFillEdgesAndRemoveNode(G, v)
5. return $\sigma$

Algorithm 2: FillCount(G)

Input: A graph $G = (V, E)$.
Output: Vertices labeled with the fill count, which is the number of edges that need to be added to make the vertex simplicial

1. foreach $v \in V$ do
2. count $\leftarrow 0$
3. foreach $v' \in \{v'|(v, v') \in E\}$ do
4. foreach $v'' \in \{v''|(v'' \neq v') \land ((v, v'') \in E)\}$ do
5. if $(v', v'') \notin E$ then
6. count $\leftarrow count + 1$
7. $fcount(v) \leftarrow count$
Algorithm 3: AddFillEdgesAndRemoveNode(G,v)

Input: A graph $G = (V, E)$, a vertex $v \in V$.
Output: A graph from which $v$ is eliminated and where the fill counts of the remaining vertices are updated.

1. foreach $v' \in \{ v'| (v, v') \in E \} \ do$
2.     foreach $v'' \in \{ v''|(v'' \neq v') \wedge ((v, v'') \in E) \} \ do$
3.         if $(v', v'') \notin E$ then
4.             foreach $x \in \{ x | (x, v') \in E \} \ do$
5.                 if $(x, v'') \in E$ then
6.                     $fcount(x) \leftarrow fcount(x) - 1$
7.                 else
8.                     $fcount(v') \leftarrow fcount(v') + 1$
9.             foreach $x \in \{ x | (x, v'') \in E \} \ do$
10.                if $(x, v') \notin E$ then
11.                   $E \leftarrow E \cup \{(v', v'')\}$
12.
13. foreach $v' \in \{ v'| (v, v') \in E \} \ do$
14.     foreach $y \in \{ y | (y \neq v) \wedge ((y, v') \in E) \} \ do$
15.         if $(y, v) \notin E$ then
16.             $fcount(v') \leftarrow fcount(v') - 1$
17. $V \leftarrow V \setminus \{v\}$
3 Pseudocode of CLIQUES and JOINTREE

The algorithm for finding the maximal cliques is called CLIQUES and borrowed from [4].

**Algorithm 4: CLIQUES(G, σ)**

**Input:** Graph $G = (V, E)$ and the perfect elimination order $σ$, where $|V| = n$.

**Output:** $Q$, list of maximal cliques

1. $Q ← ∅$
2. foreach $v ∈ V$ do $S(v) ← 0$
3. for $i = 1$ to $n$ do
   4. $v ← σ(i)$
   5. $X ← \{x ∈ Adj(v) | σ^{-1}(v) < σ^{-1}(x)\}$
   6. if $Adj(v) = ∅$ then $Q ← Q ∪ \{v}\}$
   7. if $X = ∅$ then return $Q$
   8. $u ← σ(\min\{σ^{-1}(x) | x ∈ X\})$
   9. $S(u) ← \max\{S(u), |X| - 1\}$
10. if $S(v) < |X|$ then
   11. $Q ← Q ∪ \{v\} ∪ X$
12. return $Q$

The algorithm for computing the join tree is the well-known max-cardinality algorithm, also presented in [3].

**Algorithm 5: JOINTREE(Q)**

**Input:** $Q$ a set of cliques

**Output:** A join tree $T(Q, E)$

1. $X ← ∅$
2. $E ← ∅$
3. foreach $q ∈ Q$ do
   4. $E ← E ∪ \{(q, q’)\}$ where $(q’ ∈ X)$ and $(∃q'' ∈ X, |q ∩ q''| > |q ∩ q’|)$
   5. $X ← X ∪ \{q\}$
4. return $T(Q, E)$

4 Complexity Analysis

The complexity of Algorithm 1 depends on the complexity of

- Line 1 in Algorithm 1 (FILLCOUNT). The complexity of FILLCOUNT is determined by the three nested loops: $O(n^3)$. 


• Line 3 in Algorithm 1. The complexity of this step is $\times(n)$ (list) or $O(\log n)$ (heap).

• Line 5 in Algorithm 1 (AddFillEdgesAndRemoveNode). AddFillEdgesAndRemoveNode has three nested loops, each looping over at most all the vertices of the graph. Thus, the complexity of AddFillEdgesAndRemoveNode is $O(n^4)$.

The complexity of MinFill is dominated by $n$ times the complexity of AddFillEdgesAndRemoveNode, and is thus $O(n^4)$.

The complexity of Cliques is $O(|V| + |E|)$. The loops iterate $|V|$ times. The call to Adj once for each node visits each edge at most twice, hence we add $O(|E|)$.

The complexity of JoinTree is $O(|Q|^2)$.

References


