EXPLOITING LOCAL INTERACTIONS TO BUILD GLOBAL STRATEGIES

IN HONOR OF EUGENE C. FREUDER
JONATHAN LIVINGSTON SEAGULL OF CP


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OUTLINE

- Local consistency [Freuder+ 78,82,85,96]
  - $k$-consistency, $(i,j)$-consistency, inverse consistency
- Decomposition strategies [Freuder+ 93,95]
  - Factoring Out Failure, Inferred Disjunctive Constraints
  - A general schema: disjunctive/conjunctive, properties
- Interchangeability [Freuder+ 91,95,97,05,10]
  - A theory of interchangeability: Core concepts & variations (local, weak, generalizations)
  - In multi-dimensional CSPs
Higher Consistency Levels

- $k$-consistency, $(i,j)$-consistency
  - Enforcing it may require adding constraints 😞
- Neighborhood Inverse Consistency, a $(1,j)$-consistency
  - No added constraints, no additional space needed 😊
  - Adapts to structure of constraint graph 😊
  - Expensive on dense graphs, useless on sparse graphs (same pruning as arc consistency) 😕

- Idea: Use the dual graph [2010,2011]
  - Filtering relations
  - Dense: remove redundant edges [Jégou 1989]
  - Large loops: triangulate dual graph
  - Higher levels consistency become possible!
  - Algorithm’s complexity bounded by degree of dual graph
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COMPLETE NOGOOD SETS [1993—1997]

- Consider a clique in the co-microstructure of a CSP

- Related decompositions
  - VAD: cliques efficiently computed
  - Microstructure-based decomposition
  - Inferred Disjunctive Constraints (IDC)
  - Factoring Out Failure (FOF)

- General Decomposition Schema
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INTERCHANGEABILITY

- Basic: Equivalence of 2 values for a variable
- Local form: Neighborhood Interchangeability

Dynamic Bundling [2001,2002]
- For non-binary CSPs [2003—Freuder 2005]
- For join query computation in Relational DB [2004]
**Dynamic Bundling: Advantages**

- Same operations as Forward Checking
- Bundling no-goods is amazingly effective

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**Diagram:**

- **No-good bundle:**
  - $\{1, 2\}$
  - $\{1, 3\}$
  - $\{2\}$
  - $\{3, 4\}$

- **Solution bundle:**
  - $\{1\}$
  - $\{2\}$
  - $\{1\}$

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- $V_1$: $\{1\}$
- $V_2$: $\{1\}$
- $V_3$: $\{1\}$
- $V_4$: $\{1\}$
CONCEPTS IN ORIGINAL PAPER

- Local vs Global
  - Neighborhood Interchangeability (NI)
  - Inverse $k$ Interchangeability (IKI)
  - Full Interchangeability (FI)

- Weakening
  - Substitutability (ref. dominance)
  - Partial interchangeability
  - Subproblem interchangeability

- Generalization
  - Dynamic interchangeability (ref. SBDS & SBDD)
  - Meta interchangeability
  - Functional/isomorphic interchangeability: mapping values between different variables (ref. symmetry)
Original paper inspired many researchers
In Multi-Dimensional CSPs

Meta-interchangeability on each domain dimension

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Reformulation Strategy

$P_o$: Original CSP
Uni-dimensional constraints: $\{C_1, C_2, C_3, \ldots, C_n\}$

Exploit approximate symmetries to enforce $C_1$

$P_1$: A set of reformulated CSPs
Uni-dimensional constraints: $\{C_2, C_3, \ldots, C_n\}$

Exploit approximate symmetries to enforce $C_1$

$P_2$: A set of reformulated CSPs
Uni-dimensional constraints: $\{C_3, \ldots, C_n\}$

Exploit approximate symmetries to enforce $C_1$

$P_n$: A set of reformulated CSPs
Uni-dimensional constraints: $\emptyset$

Enforce remaining constraints using some Constraint Solver
ENFORCING A CONSTRAINT

\[
N^= \oplus N^= \quad N^= \oplus S^= \\
\text{id}^= \\
S^= \oplus S^= \\
\]

\[
N^= \oplus N^= \\
V_1 \\
V_2 \\
V_3 \\
\]

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N^= \oplus S^= \quad S^= \oplus S^= \\
V_1 \\
V_2 \\
V_3 \\
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N^= \oplus N^= \\
V_1 \\
V_2 \\
V_3 \\
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\[
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V_1 \\
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Dispensability  

- Removing values, instantiations (a set of vvps)
  - Inconsistent, enforcing consistency
  - Consistent, because satisfiability is preserved
- Dispensable values, instantiations
  - Inconsistent $\Rightarrow$ Interchangeable $\Rightarrow$ Substitutable $\Rightarrow$ Removable [Bordeaux+ 08] $\Rightarrow$ Dispensable
- Ties
  - Consistency, Interchangeability, Decomposition
- That’s is all reformulation, folks!
ON A PERSONAL NOTE...

- My first presentation in grad school (1990)
  - Backtrack-free search & backtrack-bounded search
- Reason for SARA’s archival proceedings
- Hosted & mentored my students during Summer 2010
  - Lived my own dream through them
- ... A visionary, a builder, a talent ‘gatherer’
  - A single day visiting with him, Steven Prestwich, Rick Wallace, Nick Wilson, etc. is worth months of solitary study in my office
  - 4C is the largest academic group in CP, entrusted in the good hands of Barry
- My wishes to Gene
  - Lots of fun, that is, more time for research... in the US
SUMMARY

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SUMMER 2010 @ 4C