Exploiting Local Interactions To Build Global Strategies

In Honor of Eugene C. Freuder Jonathan Livingston Seagull of CP

Joint work with C. Bessiere, A. Davis, B.V. Faltings, E.C. Freuder, S. Karakashian, A. Lal, S. Prestwich, C. Reeson, A.M. Swearngin, R.J. Woodward, and others

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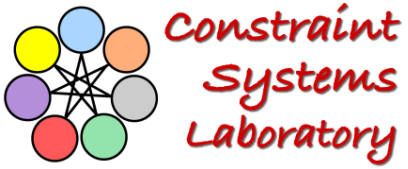

OUTLINE

- Local consistency [Freuder+ 78,82,85,96]
  - k-consistency, (i,j)-consistency, inverse consistency
- Decomposition strategies [Freuder+ 93,95]
  - Factoring Out Failure, Inferred Disjunctive Constraints
  - A general schema: disjunctive/conjunctive, properties
- Interchangeability [Freuder+ 91,95,97,05,10]
  - A theory of interchangeability: Core concepts & variations (local, weak, generalizations)
  - In multi-dimensional CSPs

Higher Consistency Levels

- k-consistency, (i,j)consistency
  - Enforcing it may require adding constraints ☹
- Neighborhood Inverse Consistency, a $(1,j)$-consistency
  - No added constraints, no additional space needed ☺
  - Adapts to structure of constraint graph ☺
  - Expensive on dense graphs, useless on sparse graphs (same pruning as arc consistency) ☹
- Idea: Use the dual graph [2010,2011]
  - Filtering relations
  - Dense: remove redundant edges [Jégou 1989]
  - Large loops: triangulate dual graph
  - Higher levels consistency become possible!
  - Algorithm's complexity bounded by degree of dual graph

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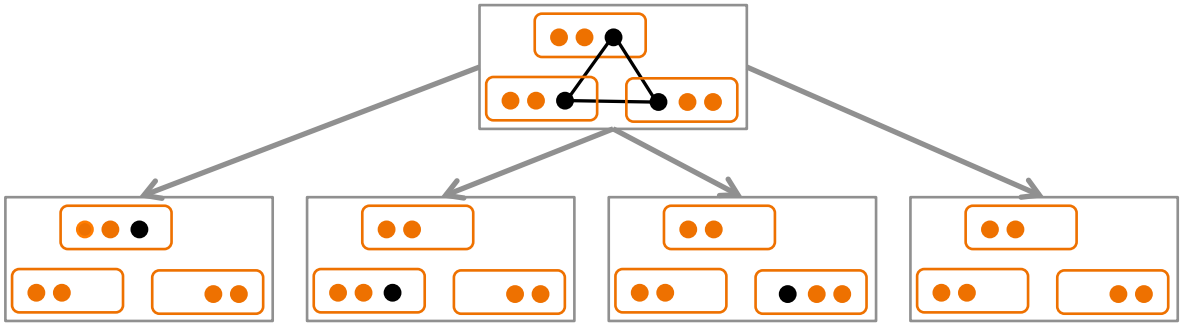
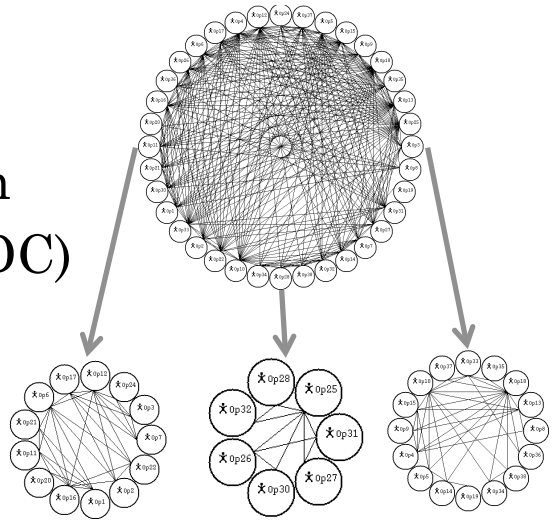
Complete NoGood Sets [1993—1997]

- Consider a clique in the co-microstructure of a CSP



- Related decompositions
  - VAD: cliques efficiently computed
  - Microstructure-based decomposition
  - Inferred Disjunctive Constraints (IDC)
  - Factoring Out Failure (FOF)
- General Decomposition Schema



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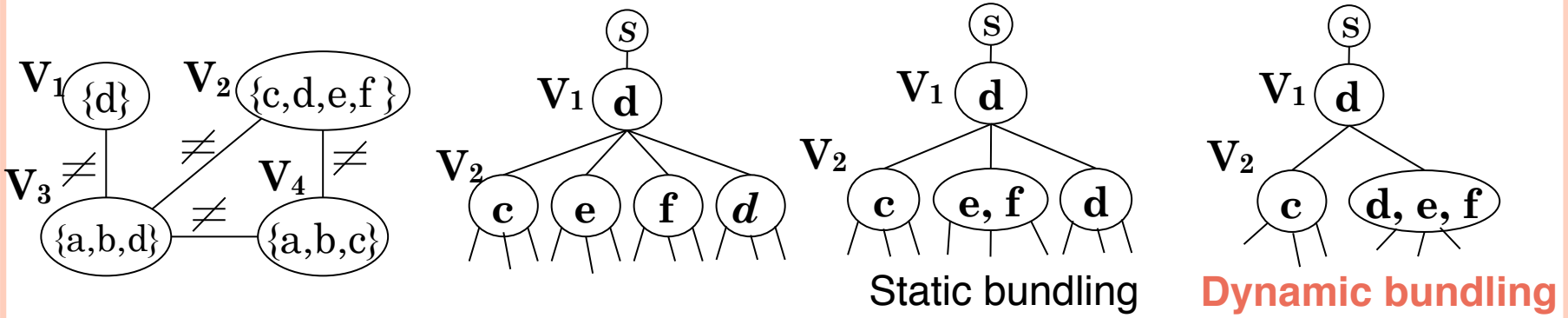
INTERCHANGEABILITY

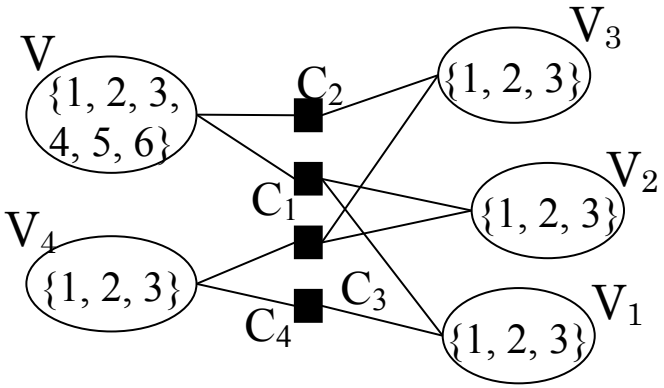
- Basic: Equivalence of 2 values for a variable
- Local form: Neighborhood Interchangeability

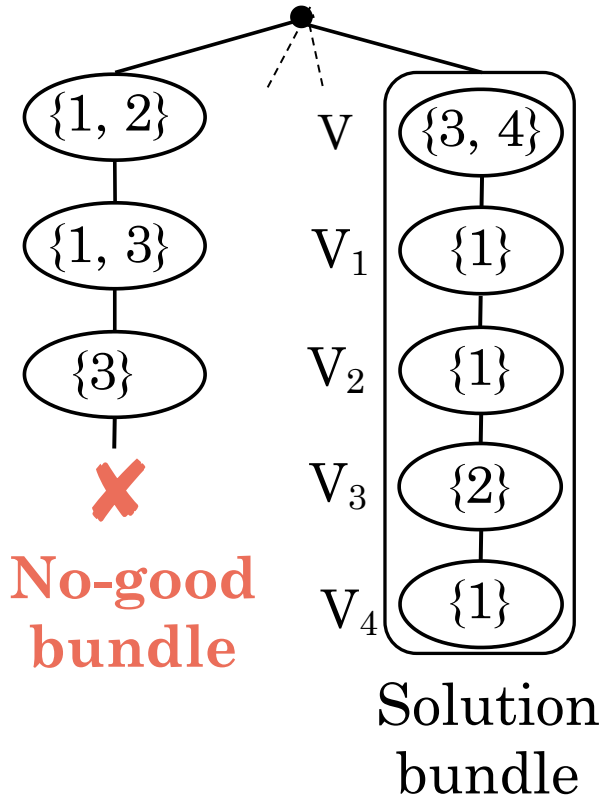


- Dynamic Bundling [2001,2002]
  - For non-binary CSPs [2003—**+Freuder 2005**]
  - For join query computation in Relational DB [2004]

Dynamic Bundling: Advantages

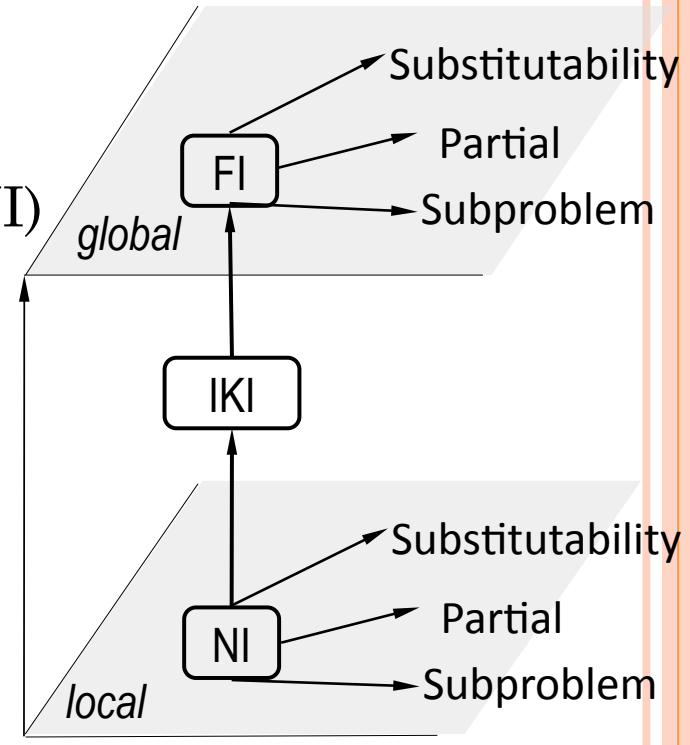
- Same operations as Forward Checking
- Bundling no-goods is amazingly effective





Concepts in Original Paper

- Local vs Global
  - Neighborhood Interchangeability (NI)
  - Inverse *k* Interchangeability (IKI)
  - Full Interchangeability (FI)
- Weakening
  - Substitutability (ref. dominance)
  - Partial interchangeability
  - Subproblem interchangeability
- Generalization
  - Dynamic interchangeability (ref. SBDS & SBDD)
  - Meta interchangeability
  - Functional/isomorphic interchangeability: mapping values between different variables (ref. symmetry)



INTERCHANGEABILITY LANDSCAPE [+Freuder 2010]

- Original paper inspired many researchers

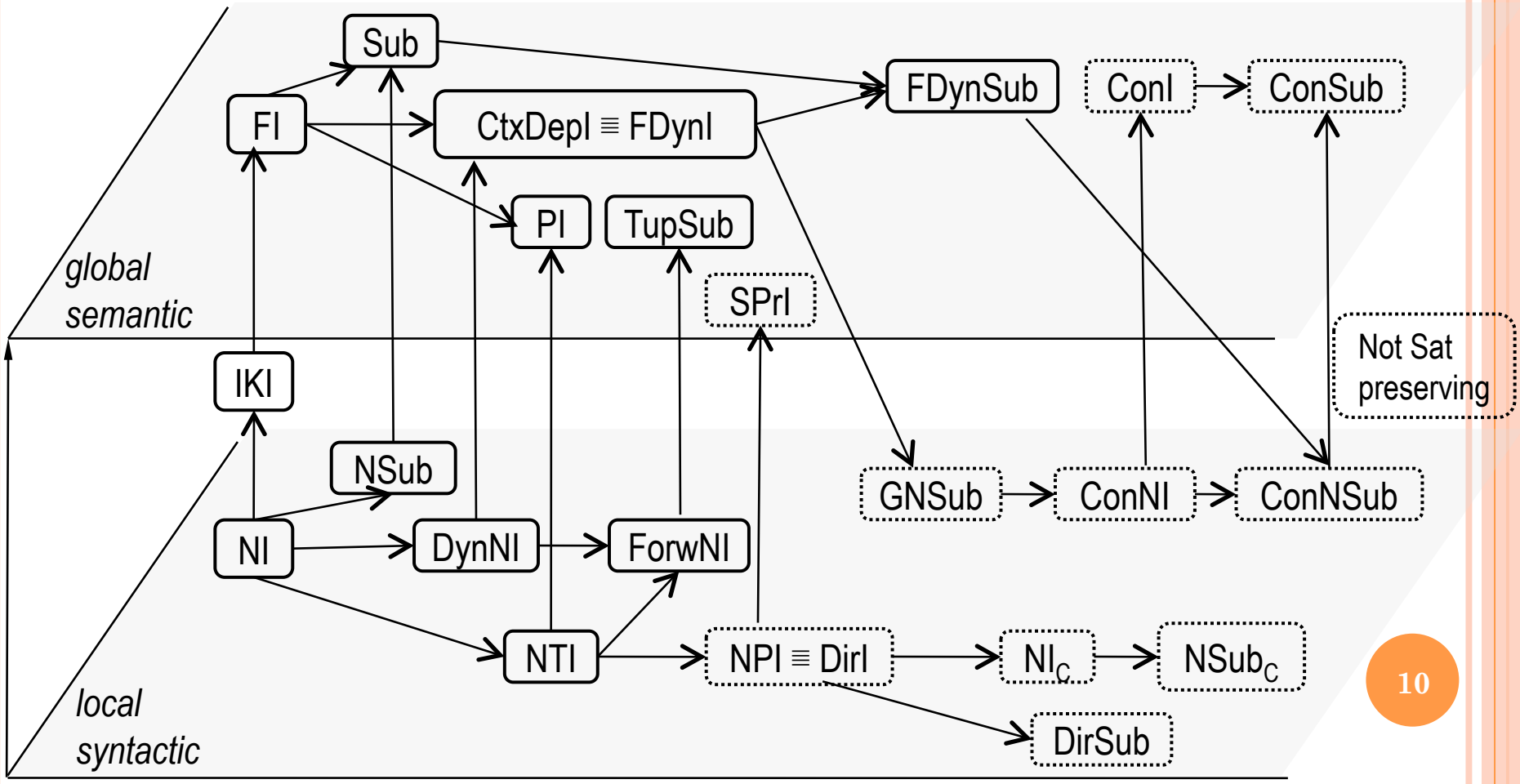
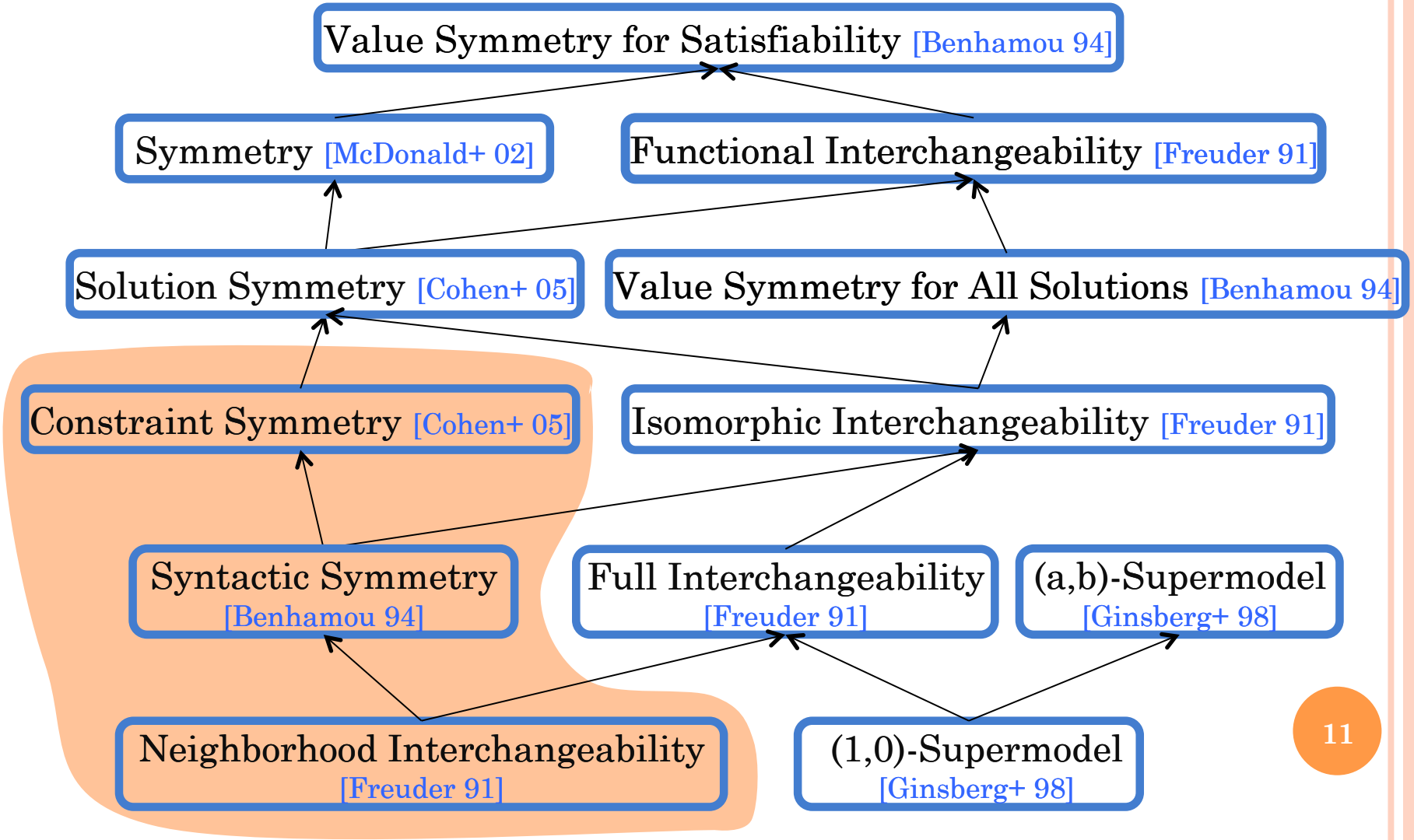


Diagram of Symmetry Concepts [+Freuder 2010]



In Multi-DimenSional CSPs [+Freuder 2011]

- Meta-interchangeability on each domain dimension

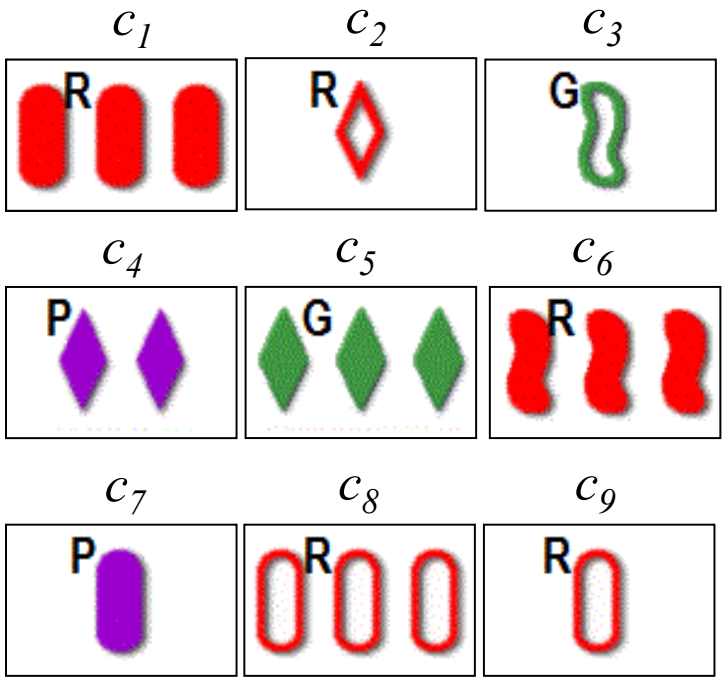
Attribute/dom	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
Number	3	1	1	2	3	3	1	3	1
Color	r	r	g	p	g	r	p	r	r
Filling	f	e	e	f	f	f	f	e	e
Shape	o	d	s	d	d	s	d	o	o

Attribute/dom	c_1	c_5	c_6	c_8	c_2	c_3	c_7	c_9	c_4
Number	3	3	3	3	1	1	1	1	2

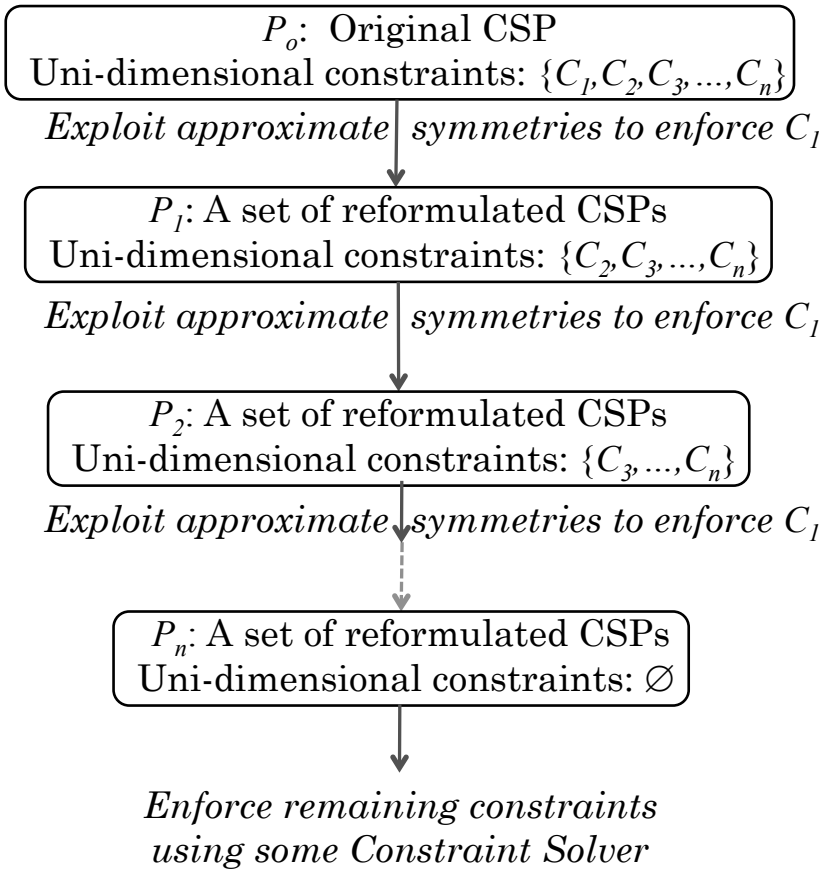
Attribute/dom	c_1	c_2	c_6	c_8	c_9	c_3	c_5	c_4	c_7
Color	r	r	r	r	r	g	g	p	p

Attribute/dom	c_1	c_4	c_5	c_6	c_7	c_2	c_3	c_8	c_9
Filling	f	f	f	f	f	e	e	e	e

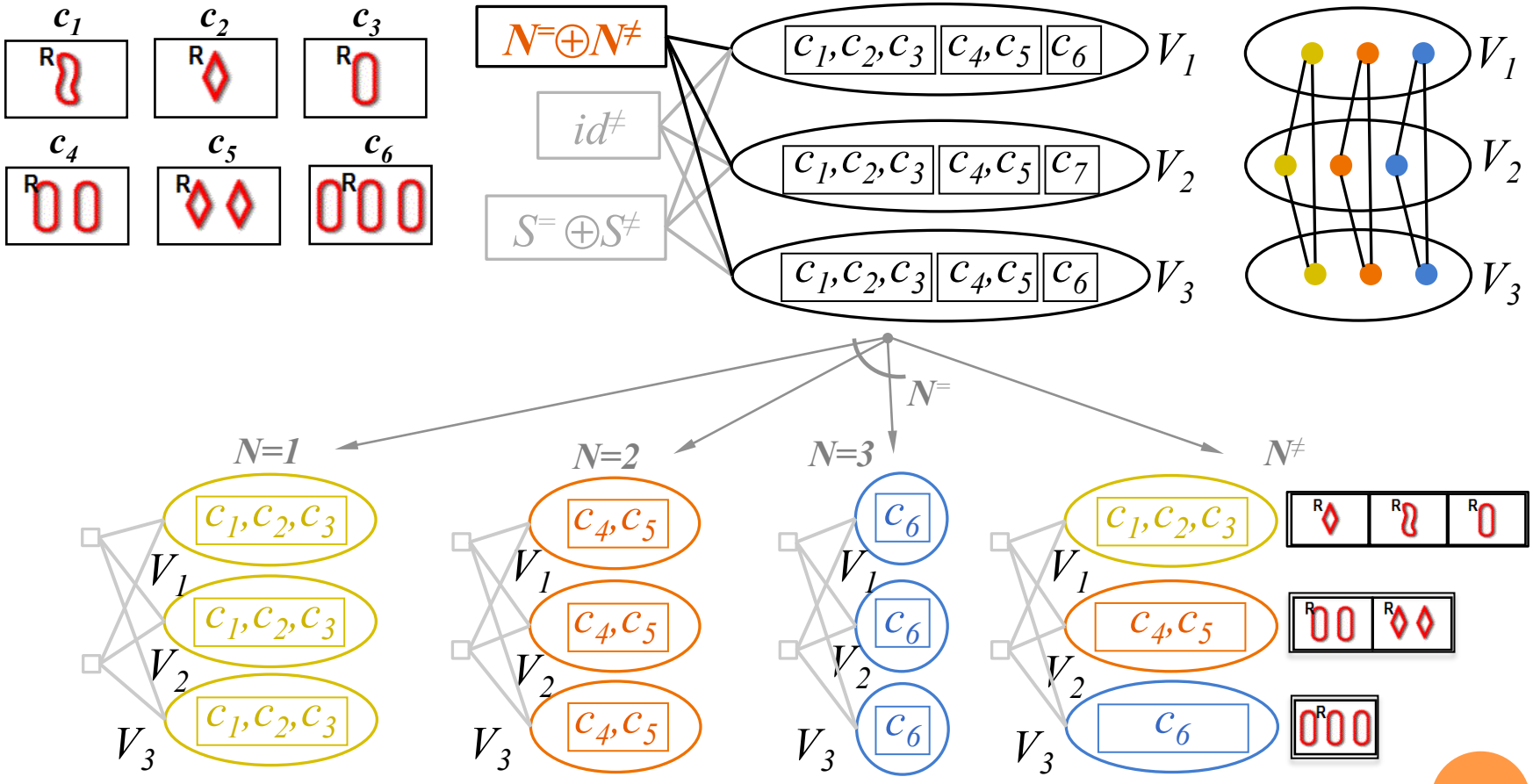
Attribute/dom	c_1	c_7	c_8	c_9	c_2	c_4	c_5	c_3	c_6
Shape	o	o	o	o	d	d	d	s	s



Reformulation Strategy



ENFORCING A CONSTRAINT



DISPENSABILITY [Freuder 2011]

- Removing values, instantiations (a set of vvps)
  - Inconsistent, enforcing consistency
  - Consistent, because satisfiability is preserved
- Dispensable values, instantiations
  - Inconsistent ⇒ Interchangeable ⇒ Substitutable ⇒ Removable [Bordeaux+ 08] ⇒ Dispensable
- Ties
  - Consistency, Interchangeability, Decomposition
- **That's is all reformulation, folks!**

On a Personal Note…

- My first presentation in grad school (1990)
  - Backtrack-free search & backtrack-bounded search
- Reason for SARA's archival proceedings
- Hosted & mentored my students during Summer 2010
  - Lived my own dream through them
- … A visionary, a builder, a talent 'gatherer'
  - A *single* day visiting with him, Steven Prestwich, Rick Wallace, Nick Wilson, etc. is worth months of solitary study in my office
  - 4C is the largest academic group in CP, entrusted in the good hands of Barry
- My wishes to Gene
  - Lots of fun, that is, more time for research… in the US

SUMMARY

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Summer 2010 @ 4C