

Revisiting Neighborhood Inverse Consistency on Binary CSPs

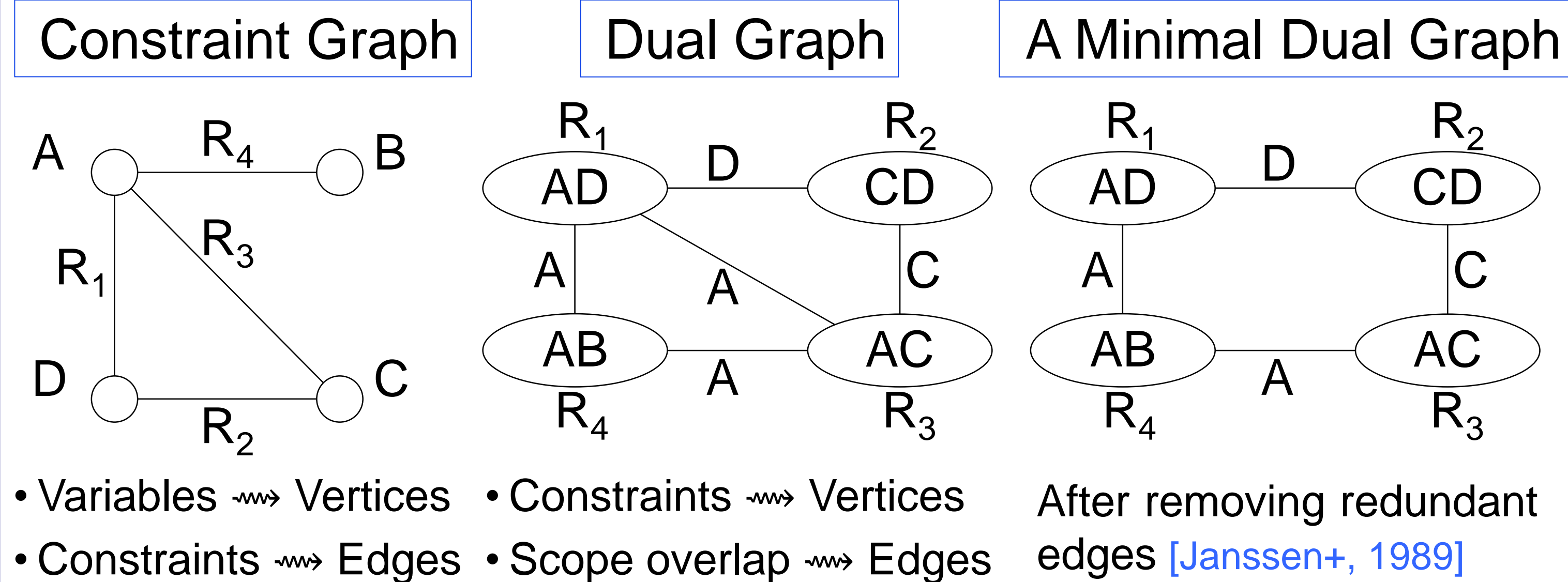
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1. Contributions

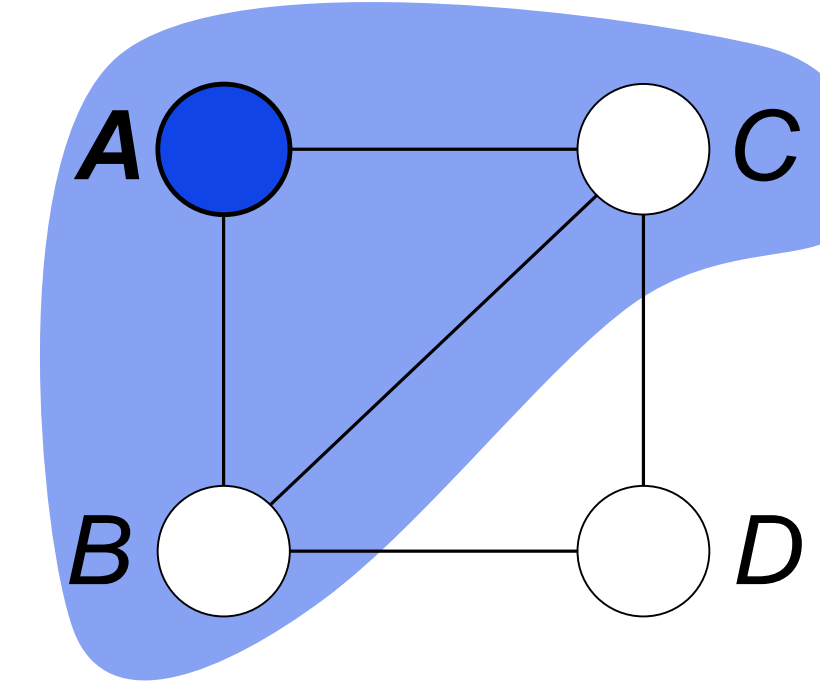
1. Understand the structure of the dual graph of a *binary* CSP
2. Determine the impact of this structure on consistency properties, e.g., NIC, sCDC, & RNIC are incomparable
3. Experimentally demonstrate the benefits of higher-level consistency

2. Graphical Representation



3. Local Consistency Properties

Neighborhood Inverse Consistency (NIC) ensures that every value in the domain of a variable can be extended to a solution in the subproblem induced by the variable and its neighborhood [Freuder & Elfe, AAI 1996]



Strong Conservative Dual Consistency (sCDC)

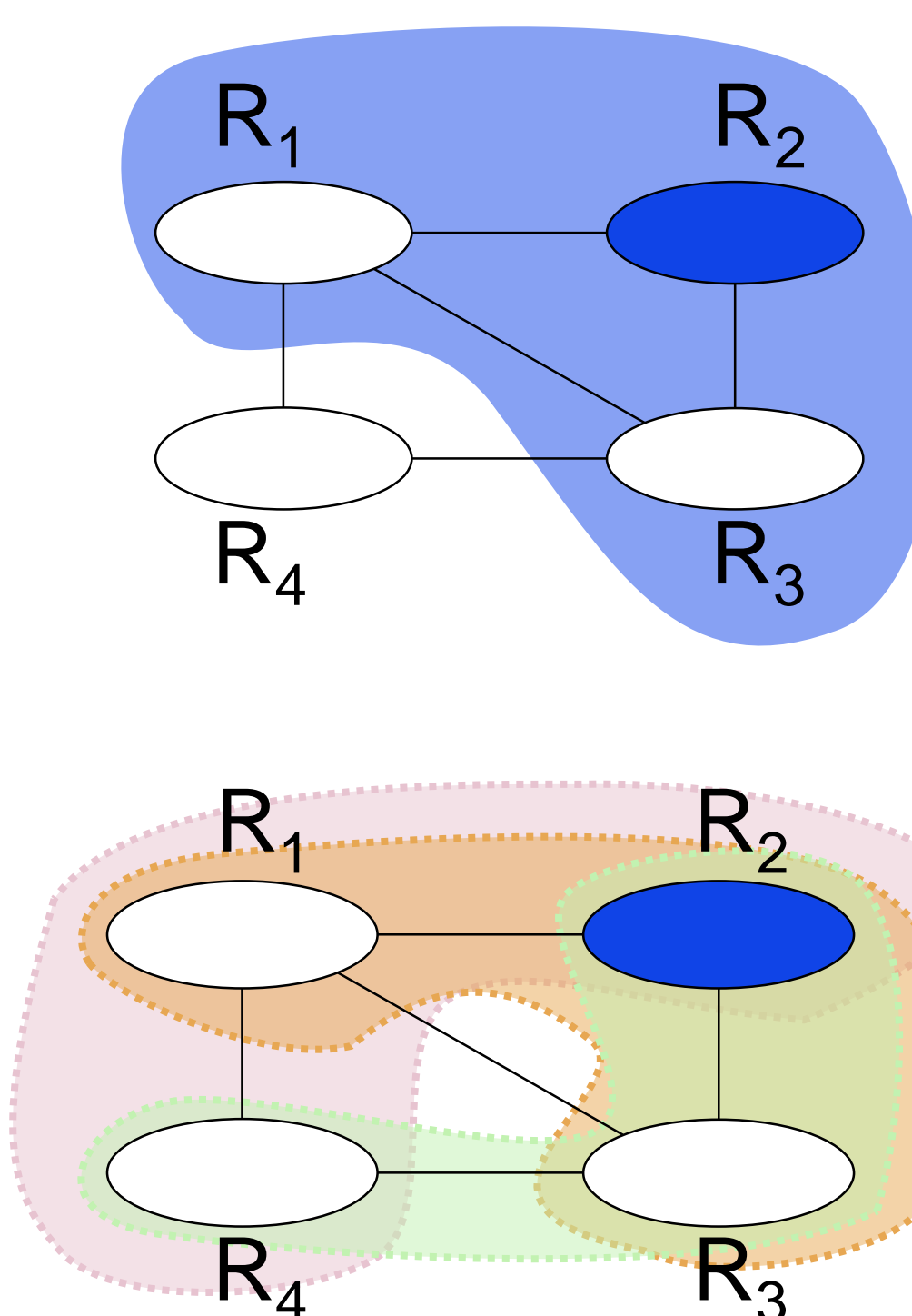
An instantiation $\{(x,a),(y,b)\}$ is sCDC iff (y,b) holds in SAC when $x=a$ and (x,a) holds in SAC when $y=b$ and (x,y) in scope of some constraint, and the problem is AC [Lecoutre+, JAIR 2011]

Relational Neighborhood Inverse Consistency (RNIC)

ensures that every tuple in every relation R_i can be extended to a solution in the subproblem induced on the dual CSP by $\{R_i\} \cup \text{Neigh}(R_i)$ [Woodward+, AAI 2011]

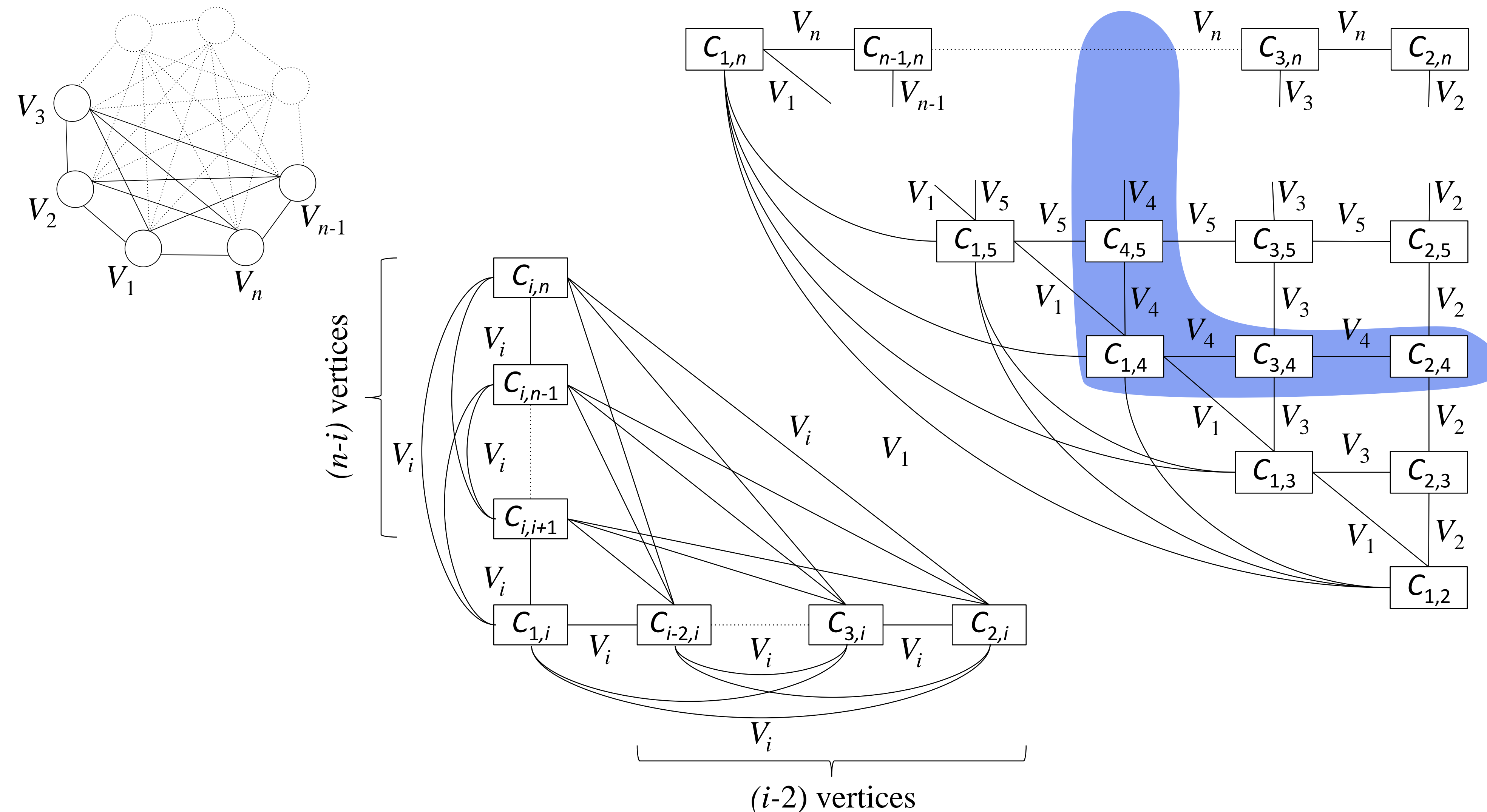
- wRNIC, triRNIC, wtriRNIC enforce RNIC on a minimal, triangulated, and minimal triangulated dual graph, respectively
- **seIRNIC** automatically selects the RNIC variant based on the density of the dual graph

$R(*,m)$ ensures that subproblem induced in the dual CSP by every connected combination of m relations is minimal [Karakashian+, AAI 2010]

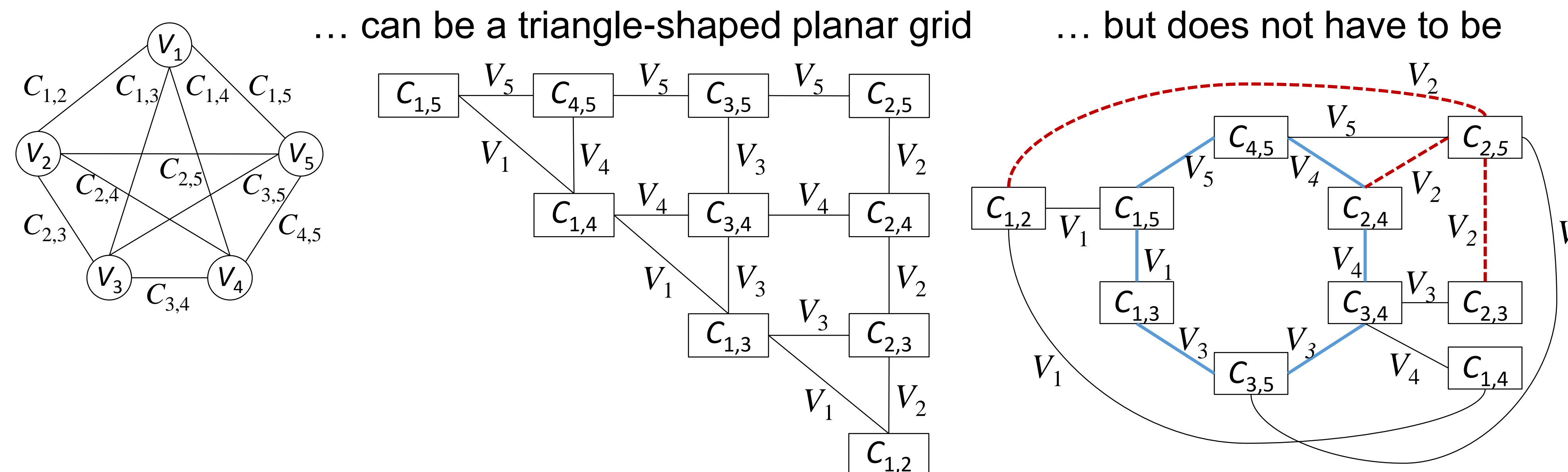


4. Structure of Dual Graph of Binary CSPs

Case1: A Complete Constraint Graph

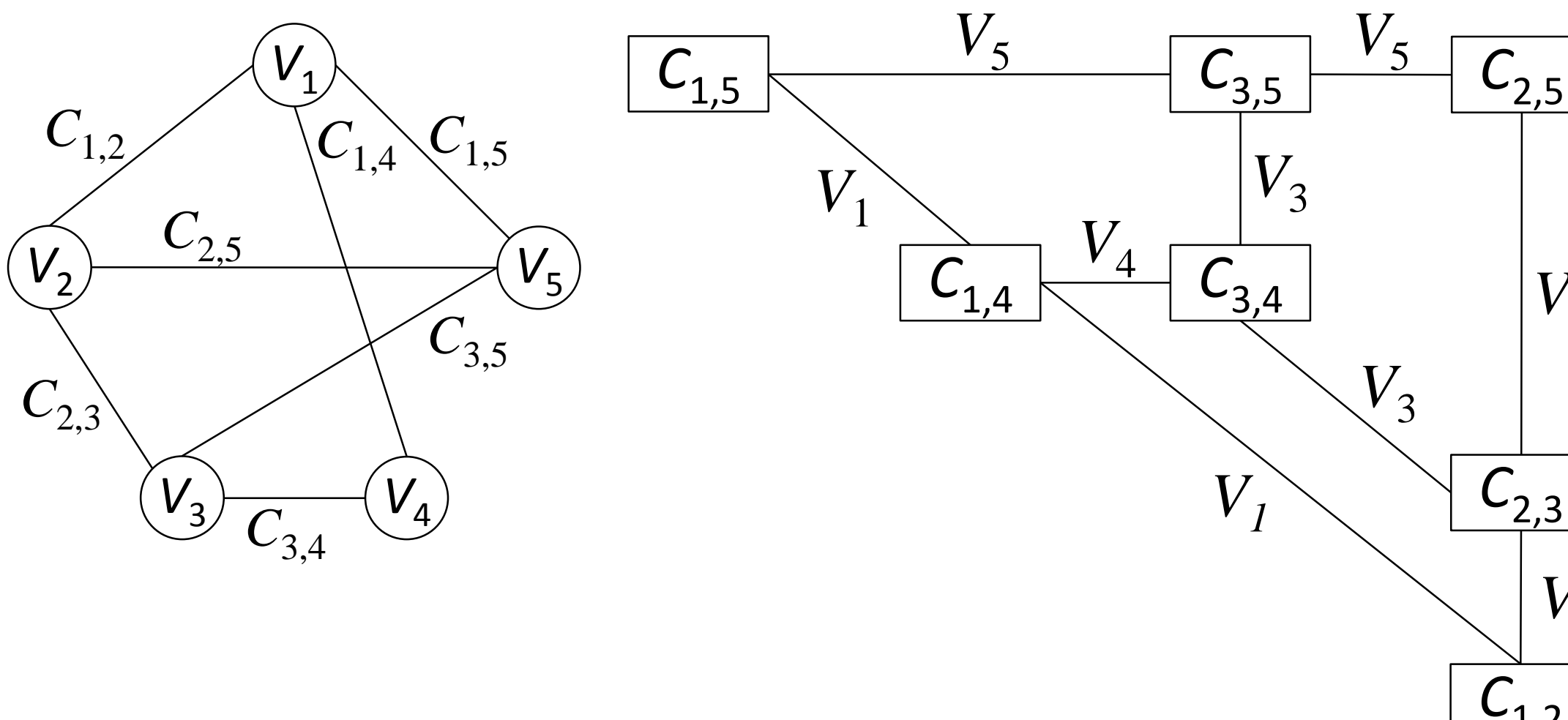


Case1: A Minimal Dual Graph of a Complete Constraint Graph



Case 2: Non-Complete Constraint Graph

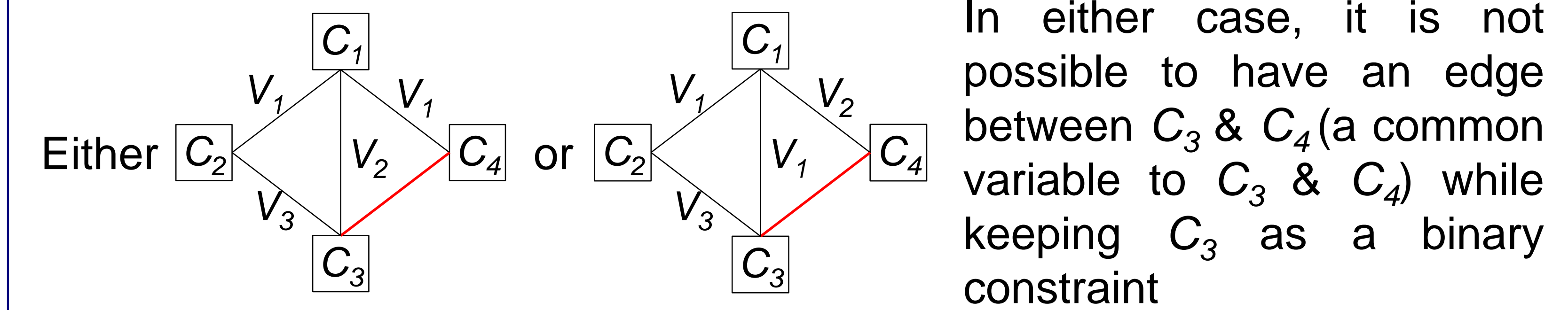
... can be a less regular grid



5. Impact on Local Consistency

wRNIC is never strictly stronger than $R(*,3)C$

wRNIC can never consider more than 3 relations simultaneously



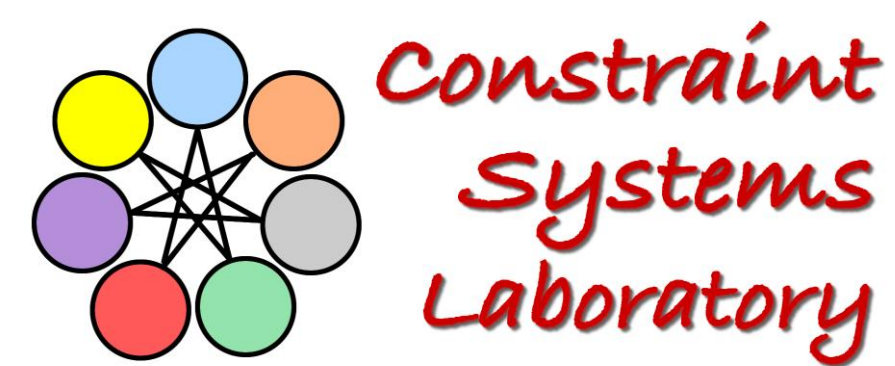
NIC, sCDC, and RNIC are incomparable

Shown using counter-examples in the paper

Experimental Results

Benchmark	# inst.	AC3.1	sCDC1	NIC	seIRNIC
CPU Time (msec)					
NIC Quickest					
bqwh-16-106	100/100	3,505	3,860	1,470	3,608
bqwh-18-141	100/100	68,629	82,772	38,877	77,981
coloring-sgb-queen	12/50	680,140	(+3) -	(+9) 57,545	634,029
coloring-sgb-games	3/4	41,317	33,307	(+1) 860	41,747
rand-2-23	10/10	1,467,246	1,460,089	987,312	1,171,444
rand-2-24	3/10	567,620	677,253	(+7) 3,456,437	677,883
sCDC1 Quickest					
driver	2/7	(+5) 70,990	(+5) 17,070	358,790	(+4) 185,220
ehi-85	87/100	(+13) 27,304	(+13) 573	513,459	(+13) 75,847
ehi-90	89/100	(+11) 34,687	(+11) 605	713,045	(+11) 90,891
frb35-17	10/10	41,249	38,927	179,763	73,119
RNIC Quickest					
composed-25-1-25	10/10	226	335	1,457	114
composed-25-1-2	10/10	223	283	1,450	88
composed-25-1-40	9/10	(+1) 288	(+1) 357	120,544	(+1) 137
composed-25-1-80	10/10	223	417	(+1) -	190
composed-75-1-25	10/10	2,701	1,444	363,785	305
composed-75-1-2	10/10	2,349	1,733	48,249	292
composed-75-1-40	7/10	(+1) 1,924	(+3) 1,647	631,040	(+3) 286
composed-75-1-80	10/10	1,484	1,473	(+1) -	397

Benchmark	# inst.	AC3.1	sCDC1	NIC	seIRNIC	AC3.1	sCDC1	NIC	seIRNIC
BT-Free									
NIC Quickest									
bqwh-16-106	100/100	0	3	8	5	1,807	1,881	739	1,310
bqwh-18-141	100/100	0	0	1	0	25,283	25,998	12,490	22,518
coloring-sgb-queen	12/50	1	0	16	1	91,853	-	15,798	91,853
coloring-sgb-games	3/4	1	1	4	1	14,368	14,368	40	14,368
rand-2-23	10/10	0	0	10	0	471,111	471,111	12	471,111
rand-2-24	3/10	0	0	10	0	222,085	222,085	24	222,085
sCDC1 Quickest									
driver	2/7	1	2	1	1	3,893	409	3,763	3,763
ehi-85	87/100	0	100	87	100	1,425	0	0	0
ehi-90	89/100	0	100	89	100	1,298	0	0	0
frb35-17	10/10	0	0	0	0	24,491	24,491	24,491	24,346
RNIC Quickest									
composed-25-1-25	10/10	0	10	10	10	153	0	0	0
composed-25-1-2	10/10	0	10	10	10	162	0	0	0
composed-25-1-40	9/10	0	10	9	10	172	0	0	0
composed-25-1-80	10/10	0	10	1	10	112	0	-	0
composed-75-1-25	10/10	0	10	10	10	345	0	0	0
composed-75-1-2	10/10	0	10	10	10	346	0	0	0
composed-75-1-40	7/10	0	10	7	10	335	0	0	0
composed-75-1-80	10/10	0	10	1	10	199	0	-	0



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